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A New Model to Determine the Dispersion of Fatigue Damage Evaluations

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Reliable predictions of remaining lives of civil or mechanical structures subjected to fatigue damage are very difficult to be made. In general, fatigue damage is extremely sensitive to the random variations of material mechanical properties, environment and loading. These variations may induce large dispersions when the structural fatigue life has to be predicted. Wirsching (1970) mentions dispersions of the order of 30 to 70 % of the mean calculated life. The presented paper introduces a model to estimate the fatigue damage dispersion based on known statistical distributions of the fatigue parameters (material properties and loading). The model is developed by expanding into Taylor series the set of equations that describe fatigue damage for crack initiation.
Keywords: Fatigue, reliability, life prediction, random loading

Introduction

A simple model to predict cumulative fatigue damage of a structure is the Palmgren-Miner's cycle-ratio summation theory (Miner, 1945). In a deterministic way, Miner's rule is written as:

$$D = \Sigma_i^P d_i = \Sigma_i^P \frac{n_i}{N_i} = n_t \cdot \Sigma_i^P \frac{f_i}{N_i} \qquad (1)$$

where d_i is the calculated damage after a total of n_i cycles of same stress (Ds$_i$ = stress range, sm$_i$ = mean stress) and N_i is the total life corresponding to each stress pair as it was the only one soliciting the point in consideration of the structure. In expression (1) n_t is the total number of cycles in the loading history and f_i is the relative frequency of a cycle of some stress (= n_i/n_t).

When n_i is a random variable and the material has strength variations such that N_i is also a random variable, it is important to determine the statistical behavior of the damage (mean, variance, maximum damage, cumulative distribution, etc). Defining the uncertainties of d_i, n_i , and N_i respectively as dd_i , dn_i , and dN_i , and assuming that these are reasonably small, dd_i can be calculated as function of dn_i and dN_i using the linear terms of a Taylor's expansion;

$$d_i + \delta d_i \approx \frac{n_i + \delta n_i}{N_i + \delta N_i} \approx \frac{n_i}{N_i} + \frac{\partial d_i}{\partial n_i} \cdot \delta n_i + \frac{\partial d_i}{\partial N_i} \cdot \delta N_i \qquad (2)$$

and the damage uncertainty will be written as

$$\delta D = \Sigma_i^P \delta d_i \qquad (3)$$

The next sections will deal with the calculation of dd_i , dn_i , and dN_i in a more explicit and rigorous way.

Nomenclature

$\hat{n}$ = average number of cycles counted for the K loading histories
$\hat{f}$ = mean relative frequency of the stress range based on field data
1 = *column vector*
2N = *number of reversion*
b = fatigue strength exponent
c = fatigue ductility exponent
d = damage increment
D = total damage
E = Young modulus
f = relative frequency of a cycle of some stress range
F = *covariance matrix*
f = *data matrix of the frequencies associated to stress histograms*
I = *identity matrix*
K = number of histograms
K' = cyclic strength coefficient
K_f = fatigue stress concentration factor
n = number of cycles applied associated to the same stress range
N = total life associated to a constant stress range
n' = cyclic strength exponent
X^0 = *vector of the average values of the fatigue and mechanical properties, of the stress range, and the fatigue stress concentration factor*

Greek Symbols

V = *vector of the mean values of the mechanical properties*
v = generic variable
D*S* = *nominal stress range*
De = *strain range*
Ds = *stress range*
G^0 = *vector of the average value of the fatigue properties and the strain range*
c^0 = *vector of the mean value of the mechanical properties and the stress range*
d*d* = *uncertainty of the damage increment*
d*D* = *uncertainty of the total damage*
d*n* = *uncertainty of the number of cycles*
d*N* = *uncertainty of the total life associated to a stress range*
e'$_f$ = *fatigue ductility coefficient*
s*m* = *mean stress*
s' = *fatigue strength coefficient*

Subscripts

f = relative to fatigue properties
t = relative to number total of cycles
ut = relative to tensile strength
y = relative to yield strength
P = relative to number of class of the histogram

Damage Calculation

The fatigue analysis of a structural point requires the characterization of two general aspects. The first aspect involves load description in terms of a typical history and its possible variation. The second aspect involves the establishment of the material fatigue constitutive equations and its variability in terms of the material fatigue parameters.

Damage Calculation - Loading Variability

Each stress history is unique and is dependent on: (*i*) minute variations of the geometry of the structural component; (*ii*) the specific variations of the loading trajectory or other loading parameters. For example, it can be observed that a structural part of a automotive vehicle suffers stress variation due to: the instantaneous dead weight of a vehicle, driving speed, environment temperature, skills of driver, etc. Therefore, recorded tapes of different runs of the same type of vehicle in the same road present a random behavior. This behavior will be broadened if other roads with different percentages of usage are incorporated in a big set of histories. In the approach suggested in this paper, each loading history is acquired and compacted in terms of a histogram through a stress or strain-cycle counting-technique such as the "rain-flow" or "pagoda" method (Matsuishi, 1968). After the application of the stress-cycle counting-technique to K loading histories, the relative frequency of the stress ranges, f (DS_i), can be estimated from these histories through equations (4.1) and (4.2) below.

$$\hat{f}(\Delta S_i) = \hat{f}_i = E[f_{ij}] = \frac{1}{K} \cdot \Sigma_{j=1}^{K} f_{ij} \tag{4.1}$$

$$f_{ij} = \frac{n_j(\Delta S_i)}{n_{t_j}} \tag{4.2}$$

where f_{ij} is the mean relative frequency of occurrences of DS_i in the K histories and $n_j(DS_i)$ is equal to the number of occurrences of the stress-range DS_i in the j<u>th</u> loading history; n_t is the total number of cycles which were counted in the j<u>th</u> loading history.

Equations 5.1 to 5.3 below quantify the variability of the typical history through the variance of the frequency of occurrence of the stress-ranges. The estimates of variance can be calculated through the following expressions:

Classical Estimator :

$$VAR[f_i] = E\left[(f_i - \hat{f}_i)^2\right] = \frac{1}{K-1} \Sigma_{j=1}^{K} \left(f_{ij} - \hat{f}_i\right)^2 \tag{5.1}$$

where $\hat{f}_i$ is the mean relative frequency of the stress-range in the i<u>th</u> class of the histogram and K is the number of the loading histories acquired.

Analytical Estimator, Bendat (1983):

$$VAR[f_i] = \frac{\hat{f}_i \cdot (1-\hat{f}_i)}{\hat{n}_t} \tag{5.2}$$

where $\hat{n}_t$ is the average of the total number of cycles counted for the K histories, $\hat{n}_t = \frac{1}{K} \cdot \sum_{j=1}^{k} n_{t_j}$.

The last expression does not consider the possible statistical dependence between the frequencies for each class in the K histograms. In these cases the variances are calculated through the expression (Mardia, 1979):

$$\mathbf{F} = \frac{1}{K} \cdot \mathbf{f}^{\mathrm{T}} \cdot \left(\mathbf{I} - \frac{1}{K} \cdot \mathbf{1}\mathbf{1}^{\mathrm{T}}\right) \cdot \mathbf{f} \tag{5.3}$$

where f is the (K x P) data matrix of the frequencies for each class (i = 1, ..., P) in the K histograms, 1 is a column vector of K ones, and I is the (K x K) identity matrix.

A typical histogram of the mean relative frequency of stress-range and of the estimates of standard deviations calculated through the equations 5.1 and 5.2 are shown in figure 1.

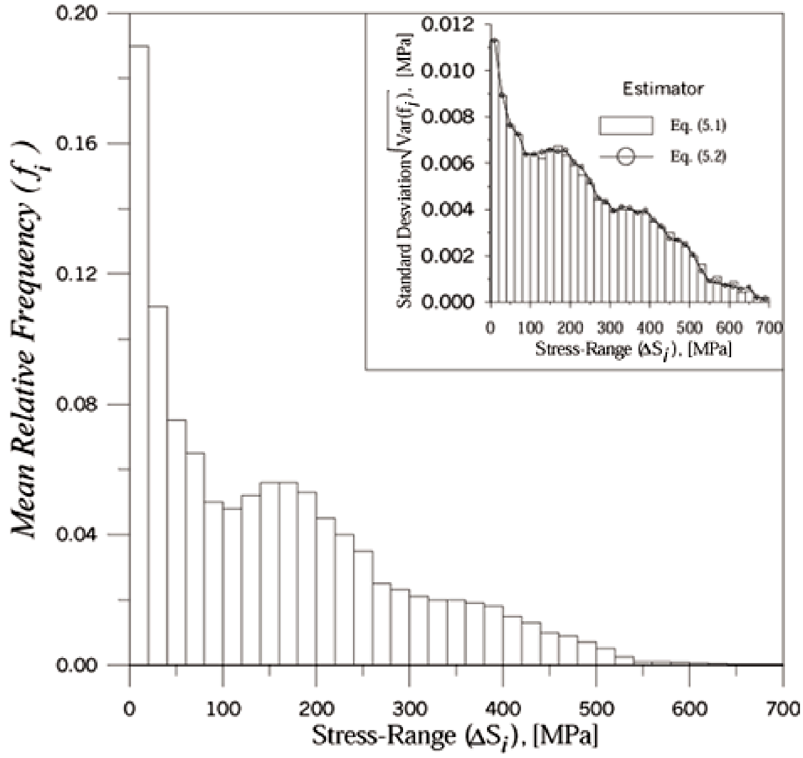

Figure 1. Typical Histograms of the Mean Relative Frequency and of the Relative Frequency Standard Deviation of the Stress-Range for a History.

Damage Calculation - Material Variability

Once the dispersion of the loading parameters, dn_i, will be evaluated by $VAR[n_i]$ or $VAR[f_i]$, the calculation of the dispersion of the accumulated damage, dD, also requires the determination of the dispersion associated with fatigue strength, dN. This section shows how the dispersions associated with the material properties influence the fatigue life N. Calculation of $VAR[N_i]$ will allow the determination of dD. The determination of $VAR[N_i]$ will be based on Coffin-Manson's ex$N$ fatigue expressions described by equations (6), (Fatigue Design Handbook, 1988).

$$\frac{(K_f \cdot \Delta S)^2}{4 \cdot E} = \frac{\Delta\sigma^2}{4 \cdot E} + \frac{\Delta\sigma}{2} \cdot \left(\frac{\Delta\sigma}{2 \cdot K'}\right)^{\frac{1}{n'}} \quad (6.1)$$

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2 \cdot E} + \left(\frac{\Delta\sigma}{2 \cdot K'}\right)^{\frac{1}{n'}} \quad (6.2)$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E} \cdot (2N)^b + e'_f \cdot (2N)^c \quad (6.3)$$

where K_f is the fatigue stress concentration factor, DS and Ds are respectively the nominal and maximum stress-range, De is the maximum strain-range; E is the Young Modulus, and K', n', s'$_f$, b, e'$_f$, and c are material fatigue properties.

Expanding equations (6.1 - 6.3) into Taylor series and approximating the mean and the variance statistics by the first order terms of the expansion, (Wirsching, 1984, Harr, 1987, Kam, 1994), it is possible to determine $E[2N]$ and $VAR[2N]$. Equations (7.1) to (7.6) show a few steps of the proposed development (Freire and Ferreira, 1995, Ferreira, 1997). Considering equation (6.1) it is possible to determine,

$$E[\Delta\sigma] = \Delta\sigma \tag{7.1}$$

$$VAR[\Delta\sigma] = \sum_m \left(\frac{\partial \Delta\sigma}{\partial v_m} \right)^2 \cdot VAR[v_m] \tag{7.2}$$

where $\frac{\partial \sigma}{\partial v_i} = \frac{\frac{\partial U(\mathbf{x}^0)}{\partial v_i}}{\frac{\partial U(\mathbf{x}^0)}{\partial (\Delta\sigma)}}$, $U(\mathbf{x}) = \frac{\Delta\sigma^2}{4 \cdot E} + \frac{\Delta\sigma}{2} \cdot \left(\frac{\Delta\sigma}{2 \cdot K'} \right)^{\frac{1}{n'}} - \frac{(K_f \cdot \Delta S)^2}{4 \cdot E}$, $\mathbf{X} = X[v_m] = [\Delta S, E, K_f, K', n']^T$,

and X^0 is the vector of the average values of the mechanical properties of the material, of the nominal stress-ranges, and the fatigue stress concentration factor.

From equation (6.2),

$$E[\Delta\varepsilon] = \Delta\varepsilon \tag{7.3}$$

$$VAR[\Delta\varepsilon] = \sum_q \left(\frac{\partial}{\partial x_q} \left(\Delta\varepsilon(\chi^0) \right) \right)^2 \cdot VAR[x_q] + 2 \cdot \sum_\lambda \frac{\partial(\Delta\varepsilon(\chi^0))}{\partial(\Delta\sigma)} \cdot \frac{\partial(\Delta\varepsilon(\chi^0))}{\partial \varsigma_\lambda} \cdot COV[\Delta\sigma, \varsigma_\lambda] \tag{7.4}$$

where c = c [x_q] = [E, K', n', Ds]^T, c^0 is the vector of the average values of the mechanical properties of the material and the stress-range, V = [E, K', n']^T, and *COV*[Ds, V$_l$] is the covariance between Ds and the material properties utilized in the Equation (6.2). This statistical measure can be estimate through of the expression:

$$COV[\Delta\sigma, \varsigma_\lambda] = \frac{\partial \Delta\sigma}{\partial \varsigma_\lambda} \cdot VAR[\varsigma_\lambda]$$

Applying the same technique to equation (6.3), but using a second order expansion, it is possible to obtain : (Freire and Ferreira, 1995; Ferreira, 1997);

$$E[2N] = 2N + \sum_j \left(\frac{\partial^2 (2N)}{\partial \upsilon_j} \cdot VAR[\upsilon_j] \right) \tag{7.5}$$

where $\frac{\partial(2N)}{\partial \upsilon_j} = \frac{\frac{\partial Q(\Gamma^0)}{\partial \upsilon_j}}{\frac{Q(\Gamma^0)}{\partial(2N)}}$, $\Gamma = \Gamma(\upsilon_j) = [E, \sigma'_f, b, \varepsilon'_f, c, \Delta\varepsilon]^T$, $Q(\Gamma) = \frac{\sigma'_f}{E} \cdot (2N)^b + \varepsilon'_f \cdot (2N)^c - \frac{\Delta\varepsilon}{2}$,

and G^0 is the vector of the average values of the fatigue properties and De. Finally,

$$VAR[2N] = \sum_j \left(\frac{\partial(2N)}{\partial\upsilon_j}\right)^2 \cdot VAR[\upsilon_j] + \\ + \frac{3}{4}\cdot\sum_j \left(\frac{\partial^2(2N)}{\partial\upsilon_j^2}\right)^2 \cdot \left(VAR[\upsilon_j]\right)^2 + \\ + \sum_{i\neq j}\left(\left(\frac{\partial^2(2N)}{\partial\upsilon_i\partial\upsilon_j}\right)^2 \cdot \left(VAR[\upsilon_i]\cdot VAR[\upsilon_j]\right)\right) + \\ + \frac{1}{2}\cdot\sum_{i\neq j}\frac{\partial^2(2N)}{\partial\upsilon_i^2}\cdot\frac{\partial^2(2N)}{\partial\upsilon^2}\cdot\left(VAR[\upsilon_i]\cdot VAR[\upsilon_j]\right) \tag{7.6}$$

where $\frac{\partial^2(2N)}{\partial\upsilon_j^2} = -\frac{\left[\frac{\partial^2 Q(\Gamma_j)}{\partial\upsilon_j^2} + \left(\frac{\partial(2N)}{\partial\upsilon_j}\right)\cdot\frac{\partial^2 Q(\Gamma)}{\partial(2N)^2}\right]}{\frac{\partial Q(\Gamma)}{\partial(2N)}} - \frac{\left[2\cdot\frac{\partial^2 Q(\Gamma)}{\partial\upsilon_j\partial(2N)}\cdot\frac{\partial(2N)}{\partial\upsilon_j}\right]}{\frac{\partial Q(\Gamma)}{\partial(2N)}}$, and

$$\frac{\partial^2(2N)}{\partial\upsilon_i\partial\upsilon_j} = -\frac{\frac{\partial^2 Q(\Gamma)}{\partial\upsilon_i\partial\upsilon_j} + \left(\frac{\partial(2N)}{\partial\upsilon_i}\cdot\frac{\partial(2N)}{\partial\upsilon_j}\cdot\frac{\partial^2 Q(\Gamma)}{\partial(2N)^2}\right)}{\frac{\partial Q(\Gamma)}{\partial(2N)}} - \frac{\frac{\partial^2 Q(\Gamma)}{\partial\upsilon_i\partial(2N)}\cdot\frac{\partial(2N)}{\partial\upsilon_j} + \frac{\partial^2 Q(\Gamma)}{\partial\upsilon_j\partial(2N)}\cdot\frac{\partial(2N)}{\partial\upsilon_i}}{\frac{\partial Q(\Gamma)}{\partial(2N)}}$$

Damage Uncertainty

The typical damage value is calculated from the mean relative frequencies of the K loading histories ($\hat{f}_i$, eq. (4.1)) and the typical life values, N_i, determined by applying the average values of loading and material properties to equation 6.3.

$$D_{Typical} = \Sigma_i^P d_i = \Sigma_i^P \frac{n_i}{N_i} = n_t \cdot \Sigma_i^P \frac{f_i}{N_i} \tag{8.1}$$

where n_t is the total number of cycles per block.

Using first and second order expansion, the mean damage can be calculated, respectively, through expressions (8.2) and (8.3).

$$E[D] = \hat{D} = \hat{n}_t \cdot \Sigma_i^P \frac{\hat{f}_i}{\hat{N}_i} \tag{8.2}$$

$$E[D] = \hat{D} = \hat{n}_t \cdot \Sigma_i^P \left(\frac{\hat{f}_i}{\hat{N}_i} + \frac{\hat{f}_i}{\hat{N}_i^3}\cdot VAR[N_i]\right) \tag{8.3}$$

where $\hat{f}_i$ and $\hat{N}_i$ and $VAR[N_i]$ were defined respectively by expressions 4.1, 7.5 and 7.6.

Expanding the equation (1), linearly using a Taylor's series, the uncertainty of damage may be represented by the variance of damage, $VAR[D]$, given by

$$VAR[D] = (\hat{n}_t)^2 \cdot \left(D_f \cdot \mathbf{F} \cdot D_f^T + D_N \cdot V_N^T \cdot V_N \cdot D_f^T\right) \tag{8.4}$$

where $V_N = [s[N_1], s[N_2], \ldots, s[N_P]]^T$, $s[\cdot]$ is the standard deviation, and D_f, D_N and F are given below by equation 8.5, 8.6 and 8.7.

$$D_f = \left[\frac{\partial D}{\partial f_1} \quad \frac{\partial D}{\partial f_2} \quad \mathrm{L} \quad \frac{\partial D}{\partial f_P} \right]^T, \quad \frac{\partial D}{\partial f_i} = \frac{1}{N_i}; \qquad (8.5)$$

$$D_N = \left[\frac{\partial D}{\partial N_1} \quad \frac{\partial D}{\partial N_2} \quad \mathrm{L} \quad \frac{\partial D}{\partial N_P} \right]^T, \quad \frac{\partial D}{\partial N_i} = -\frac{f_i}{N_i^2}; \qquad (8.6)$$

$$\text{and} \quad \mathbf{F} = \begin{bmatrix} VAR[f_1] & \cdots & COV[f_1, f_P] \\ \vdots & \ddots & \vdots \\ COV[f_P, f_1] & \cdots & VAR[f_P] \end{bmatrix} \qquad (8.7)$$

Numerical Results

This section show the results determined using the proposed model for damage uncertainty estimation and their comparison with the direct application of the Monte Carlo Method (Harr, 1987) to the same data conditions. In this analysis random variations were considered to be present in the loading histories and in the mechanical properties of the material. Damage uncertainty was generated by the combination of the variations of the loading history and material properties. The mechanical properties of the material MANTEN steel are given in the table 1.

Table 1. Mechanical Properties of MANTEN (Fatigue Design Handbook, 1988).

Monotonic Properties :	Unit	Value
Young Modulus, E	[GPa]	206
Yield Strength, S_y	[MPa]	324
Tensile Strength, S_{ut}	[MPa]	565
Cyclic Properties :		
Cyclic Strength Coefficient, K'	[MPa]	1200
Cyclic Strength Exponent, n'		0.20
Fatigue Strength Exponent, b		0.26
Fatigue Ductility Exponent, c		-0.47
Fatigue Strength Coefficient, σ'_f	[MPa]	917
Fatigue Ductility Coefficient, ε'_f		-0.095

It was assumed that the mechanical properties of the MANTEN steel presented coefficients of variation of the order of 7,5%. In other words, it was assumed that all material constants had standard deviations equal to 7,5% of their mean values. It was also assumed the presence of a notch with a stress concentration factor constant and equal to 3.

In order to evaluate the proposed model, 18 different loading histories represented by their one-sided power spectral density (PSD) were used. Each one of these 18 PSDs were used to generate 400 loading blocks through Gaussian simulation (Ferreira and Freire, 1995), each block containing 3,000 extremes (picks and valley). In this way, histograms with the estimates of the mean relative frequency and their respective uncertainties, calculated through equations (4.1) and (4.2), were based on about 1,200,000 extremes for each of the 18 different histories.

The material and loading generated above (stress-range histograms and material's properties) were used to calculate damage results through expressions 8.2 - 8.4. The estimates calculated from these equations were compared with estimates generated from the application of a Monte Carlo technique.

To infer the statistical properties of the accumulated damage through the Monte Carlo techniques it was also necessary to define the average and variance of the histogram of the stress-ranges and of the mechanical properties of the material. In this specific case, the

Monte Carlo technique was applied to the set of equations (6.1) to (6.3). The number of trials accomplished to evaluate the statistical properties of the accumulated damage were superior to 60,000. The estimates obtained from the proposed analytic model and from the Monte Carlo technique for the mean, median, standard deviation, and coefficient of variation of the damage are presented and compared respectively in the plots of figures 2 to 6.

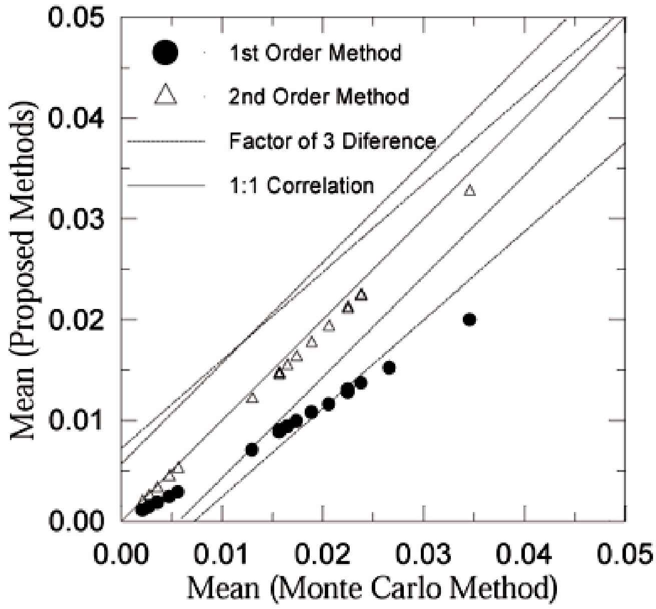

Figure 2. Mean Damage Estimates.

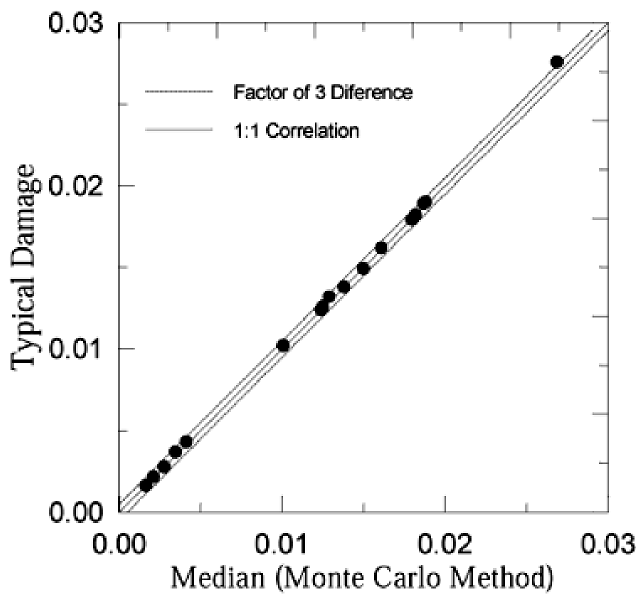

Figure 3. Median Damage Estimates.

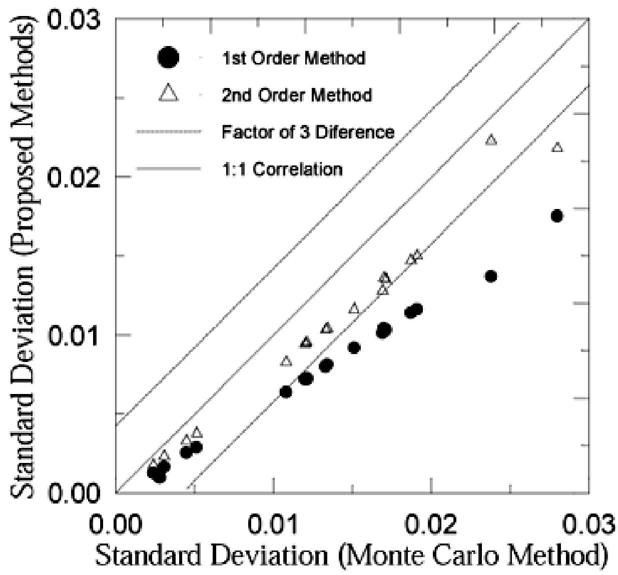

Figure 4. Damage Standard Deviation

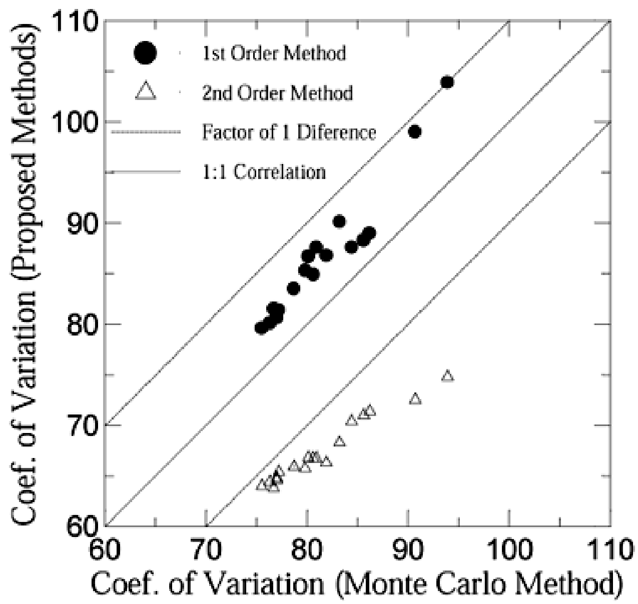

Figure 5. Coefficient of Variation of Damage.

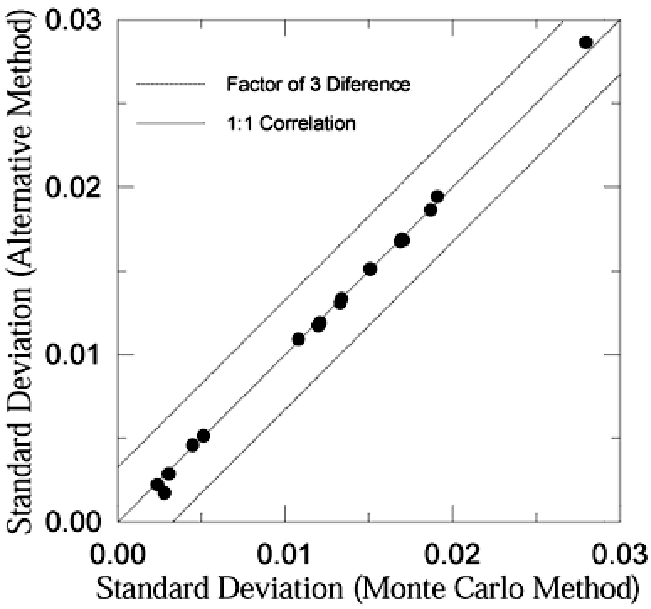

Figure 6. Standard Deviation of Damage.

The comparison between the estimates of the mean of the damage obtained analytically and through the Monte Carlo simulation are presented in figure 2. It is verified that the results obtained through the second order method, eq. (8.3), allow a quite precise evaluation of this statistic.

The comparison between the estimates of the median values of the damage, obtained by Monte Carlo simulation, with the typical damage, calculated through the equations 6.1 - 6.3, show a very good correlation as it can be observed in the figure 3. This is an interesting result because it allows the evaluation of another statistic that describes the damage behavior, in a very easy way.

The predictions of the standard deviation of the damage show that the first and second order methods presented biased estimates when compared to the respective results obtained by simulation of Monte Carlo, as it can be observed in figure 4.

However, as it can be observed in figure 5, the biased behavior of the damage dispersion is attenuated when the coefficient of variation of the damage is calculated using the analytical estimates of first order of the standard deviation and of the mean.

Using the results of the figures 4 and 5 a quite efficient form of evaluating in an unbiased way the standard deviation is to consider the product of the coefficient of variation and the mean value of the damage, calculated respectively through the equations of first and of second order. A comparison between the results obtained through this way and that calculated through the Monte Carlo method is presented in the figure 6.

Conclusions

This paper describes a set equation to determine the basic statistical parameters of fatigue damage evaluations. The predictions equations are based on Palmgren-Miner's rule and the e-N method. It allows for the combined use of random loading and random material properties. The developed model was applied to 18 damage examples and the results obtained have been compared satisfactorily with others determined by standard Monte Carlo prediction techniques.

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