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Improved Determination of Floquet's Reference Frame for the Phase-to-Ground Short Circuit of a Nonsalient-Pole Damped Alternator

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I. Introduction

THE STUDY OF the constant-speed asymmetrical operation of an unsaturated synchronous machine is of great importance. It always leads to a linear differential equation with periodic time-dependent coefficients. Very little is known about such mathematical problems, except for a theorem due to Floquet [1]–[3], which proves the existence of a transformation leading to the complete solution but does not give any general way to find that transformation. However, in some particular cases [5], [6], it has been possible to find that transformation. Note that we have discarded the Carson method [8], [9], which we have found too complicated because it introduces many time constants which are not relevant to the problem.

The method employed has a relatively general significance, and may be used in any case of linear periodic coefficient equations but it involves very tedious algebraic manipulations. Therefore, it has been necessary to address the problem anew, with matrix notations, to arrive at more general approaches to the determination of the Floquet reference frame.

The problem which is tackled here is the sudden single-phase short circuit of an unloaded damped nonsalient alternator. The use of matrix notation will be shown to lead to a solution much more straightforward then in [5] or [6].

II. Problem Description

The alternator consists of six windings, only four of which are considered here. There is one winding (f) on the rotor, called the field winding, which is assumed to be connected, through sliding contacts, to a nonimpedant dc source. Two damper windings are located on the rotor; the so-called quadrature axis damper (kq) has no mutual inductance with the field winding, while the direct axis damper (kd) has a mutual inductance with the field winding, but not with the quadrature damper. Three armature windings are located on the stator, but only one, called (a), which will be suddenly short-circuited at time 0, is to be considered here (Fig. 1). While resistances and self-inductances are constant, due to air gap uniformity, the mutual inductance between a and rotor windings varies periodically with rotor–stator displacement. In machines of practical interest, these variations are sinusoidal.

$$R\cdot I+\frac{d}{dt}(L\cdot I)=E \tag{1}$$

$$R=\begin{vmatrix} R_a & 0 & 0 & 0\\ 0 & R_f & 0 & 0\\ 0 & 0 & R_{kd} & 0\\ 0 & 0 & 0 & R_{kq}\end{vmatrix}$$

$$L=\begin{vmatrix} L_a & M_{af}\cos\theta & M_{akd}\cos\theta & M_{akq}\sin\theta\\ M_{af}\cos\theta & L_f & M_{fkd} & 0\\ M_{akd}\cos\theta & M_{fkd} & L_{kd} & 0\\ M_{akq}\sin\theta & 0 & 0 & L_{kq}\end{vmatrix}$$

$$I=\begin{vmatrix} i_a\\ i_f\\ i_{kd}\\ i_{kq}\end{vmatrix}\qquad E=\begin{vmatrix}0\\ E\\ 0\\ 0\end{vmatrix}$$

where $d\theta/dt=\omega$ is the rotor speed, and $\theta=\omega t-\Psi$ if uniform speed is considered.

When the problem is stated in these terms, saturation cannot be taken into account. However, experience shows that in most practical machines, the results obtained with

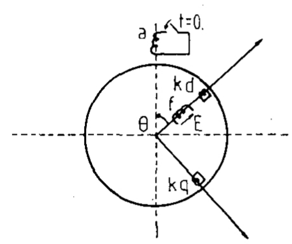

Fig. 1. Definition of the single-phase short circuit of an alternator.

this restriction may be of considerable interest; for example, the three-phase symmetrical case is almost always analyzed in linear conditions.

III. Floquet's Theorem and Application

The solution of (1) may be divided into two parts.

Floquet's theorem is indirectly used to solve the homogeneous differential equation with periodic time-dependent coefficients. Let X be the solution of the homogeneous equation; then

$$\frac{d}{dt}X=M_1\cdot X. \tag{2}$$

Let M_2 be a square matrix whose coefficients are periodic, with the same period as the coefficients of M_1:

$$M_2 = \begin{vmatrix} f_{1a}(\theta) & f_{2a}(\theta) & f_{3a}(\theta) & f_{4a}(\theta) \\ f_{1f}(\theta) & f_{2f}(\theta) & f_{3f}(\theta) & f_{4f}(\theta) \\ f_{1kd}(\theta) & f_{2kd}(\theta) & f_{3kd}(\theta) & f_{4kd}(\theta) \\ f_{1kq}(\theta) & f_{2kq}(\theta) & f_{3kq}(\theta) & f_{4kq}(\theta) \end{vmatrix} \quad (3)$$

or, more compactly,

$$M_2 = |F_1(\theta) \quad F_2(\theta) \quad F_3(\theta) \quad F_4(\theta)| \quad (4)$$

where

$$F_h(\theta) = \sum_{n=-\infty}^{\infty} \bar{I}_{nh} \exp(jn\theta) \quad (5)$$

for $h = 1, 2, 3,$ and 4.

M_2 may be either real or complex. The change of variables

$$X = M_2 \cdot \bar{X} \quad (6)$$

yields

$$\frac{d}{dt}\bar{X} = M_3 \cdot \bar{X}. \quad (7)$$

Floquet's theorem states that there exists a matrix M_2 such that M_3 be constant. In addition, if all eigenvalues of M_3 are distinct, then it is possible to modify M_2 in such a way as to diagonalize M_3:

$$M_3 = \begin{vmatrix} -\alpha_1 & & & \\ & -\alpha_2 & & \\ & & -\alpha_3 & \\ & & & -\alpha_4 \end{vmatrix}. \quad (8)$$

In what follows, we shall assume that this is so although it is not strictly impossible that two time constants of a synchronous machine are equal. Indeed, the probability of such an event is practically zero under any circumstance we have ever heard of.

As already stated, no general method to determine M_2 is available. However, the important point is that Floquet's theorem shows that the solution of (2) can be written as

$$X = \sum_{h=1}^{4} K_h \exp(-\alpha_h \cdot t) \cdot F_h(\theta) \quad (9)$$

where the K_h's are constant.

As another consequence of Floquet's theorem, one solution of the inhomogeneous equation is a vector $F_0(\theta)$, which can be expanded in a Fourier series:

$$F_0(\theta) = \sum_{n=-\infty}^{\infty} \bar{I}_{n0} \exp(jn\theta). \quad (10)$$

Thus, as

$$I = X + F_0(\theta) \quad (11)$$

the solution of (1) can be written

$$I = \sum_{h=0}^{4} K_h \exp(-\alpha_h \cdot t) \cdot F_h(\theta) \quad (12)$$

where $\alpha_0 = 0$ and $K_0 = 1$.

Our method can be described as follows.

1) Substitute (12) into (1).
2) Establish recurrent relation between the vectors $\bar{I}_{nh}$'s.
3) Realize that those relationships are generally not convergent; the definition of convergence provides, in fact, the equation to determine the α_h's in (8).
4) Use the values of α and the recurrence relations to find the vectors $F_h(\theta)$, called Floquet's reference frame.
5) Use four initial conditions to determine completely the currents.

IV. The Recurrent Relation

Let us introduce (12) into (1), and then let us cancel the groups of terms which are multiplied by the same $\exp(-\alpha_h \cdot t + jn\theta)$; since it is possible to treat independently the different values of h, we shall omit the index h from here to the end of the next section. This yields

$$A \cdot \bar{I}_{n+1} + B_n \bar{I}_n + C \cdot \bar{I}_{n-1} = 0 \quad (13)$$

where

$$A = \begin{vmatrix} 0 & \sqrt{1-\sigma_{af}} & \sqrt{1-\sigma_{akd}} & j\sqrt{1-\sigma_{akq}} \\ \sqrt{1-\sigma_{af}} & 0 & 0 & 0 \\ \sqrt{1-\sigma_{akd}} & 0 & 0 & 0 \\ j\sqrt{1-\sigma_{akq}} & 0 & 0 & 0 \end{vmatrix}$$

$$C = A^*$$

$$B_n = \frac{1}{jn\omega - \alpha} \begin{vmatrix} \delta_a & 0 & 0 & 0 \\ 0 & \delta_f & 0 & 0 \\ 0 & 0 & \delta_{kd} & 0 \\ 0 & 0 & 0 & \delta_{kq} \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \sqrt{1-\sigma_{fkd}} & 0 \\ 0 & \sqrt{1-\sigma_{fkd}} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}.$$

The δ's are the reciprocals of the time constants of the four circuits, and the σ's are the dispersion coefficients between every possible pair of windings.

V. Determination of α and M_2

If a value of α were known, knowledge of any two successives vectors $\bar{I}_n$ and $\bar{I}_{n+1}$ would permit knowledge of all vectors $\bar{I}_n$ for $-\infty < n < \infty$; since (5) must be a true Fourier expansion, $\bar{I}_n$ should go to zero when n goes to plus or minus infinity. Therefore, the fundamental property of the α's in (13) is to render (5) convergent for both infinite positive and infinite negative values of n. Expressing this condition will give an equation to determine the α's without knowing the change of variables M_2. We shall develop this concept.

Let us consider first $n \geqslant 0$, and let

$$\bar{I}_{n+1} = P_{n+1} \cdot \bar{I}_n. \tag{14}$$

Substituting this definition into (13) yields

$$\{A \cdot P_{n+1} \cdot P_n + B_n \cdot P_n + C\} \cdot \bar{I}_{n-1} = 0 \tag{15}$$

and since $\bar{I}_n$ must be nonzero,

$$P_n = -\{A \cdot P_{n+1} + B_n\}^{-1} \cdot C. \tag{16}$$

The limit value P_∞ fulfills

$$P_\infty = -\{A \cdot P_\infty + B_\infty\}^{-1} \cdot C \tag{17}$$

an equation which is easily solved by successive approximations if one begins with $P_\infty = 0$ as a first guess.

Similarly, for $n \leqslant 0$, let

$$\bar{I}_{n-1} = N_{n-1} \cdot \bar{I}_n \tag{18}$$

which implies from (13)

$$N_n = -\{C \cdot N_{n-1} + B_n\}^{-1} \cdot A. \tag{19}$$

And $N_{-\infty}$ is determined in a manner similar to P_∞.

For $n = 0$, eq. (13) yields (if $h \neq 0$)

$$\{A \cdot P_1 + B_0 + C \cdot N_{-1}\} \cdot \bar{I}_0 = 0 \tag{20}$$

and since $\bar{I}_0$ must be nonzero, this implies, first, that

$$\det\{A \cdot P_1 + B_0 + C \cdot N_{-1}\} = 0 \tag{21}$$

and, second, that $\bar{I}_0$ must lie along a well-defined direction. Thus, α can be called an eigenvalue and $\bar{I}_0$ an eigenvector of the problem.

Practically, we choose a value J of n such that N_{-J} and P_J are "close enough" to $N_{-\infty}$ and P_∞. Then, we choose a first guess $\alpha^{(1)}$ for α; thus, we are able to evaluate first guesses of N_{-1} and P_1 which may be written $N_{-1}(\alpha^{(1)}, J)$ and $P_1(\alpha^{(1)}, J)$. J is "large enough" if N_{-1} and P_1 practically do not vary if J is change for $J+1$ or $J-1$.

Then, a second guess $\alpha^{(2)}$ will be found from (21) as

$$\det\{A \cdot P_1(\alpha^{(1)}, J) + B_0(\alpha^{(2)}) + C \cdot N_{-1}(\alpha^{(1)}, J)\} = 0. \tag{22}$$

In all the case which we have studied, repeating this process leads to a very fast convergence.

The problem of finding a first guess is solved by assuming that P_1 and N_{-1} are equal to, respectively, P_∞ and $N_{-\infty}$, that is, by solving

$$\det\{A \cdot P_\infty + B_0(\alpha^{(1)}) + C \cdot N_{-\infty}\} = 0. \tag{23}$$

Therefore, it is possible to compute the four eigenvalues α_h ($h = 1,2,3,4$), and to evaluate the corresponding eigenvectors $\bar{I}_{0h}$.

Once the α_h's are calculated, all the matrices P_{nh} and N_{nh} are defined by (16) and (19). The directions of all the vectors $\bar{I}_{nh}$ are given by (14) and (18). Then we find the directions of the four vectors $F_h(\theta)$ which are the columns of the matrix M_2. Since the $F_h(\theta)$'s are, in fact, the axis of the new reference frame which transforms the system of differential equations with periodic coefficients (2) into another system with constant coefficients (7), it can be said that Floquet's reference frame is completely determined.

VI. Evaluation of Currents

To compute the currents, we first have to find the particular solution (10). This is done by substituting (10) into (1) for $n = 0$. From that we find

$$\bar{I}_{00} = \begin{vmatrix} 0 \\ E/R_f \\ 0 \\ 0 \end{vmatrix} \tag{24}$$

with the same recurrent relation, and with $\alpha = 0$, we can now find all the other vectors $\bar{I}_{n0}$ and the solution $F_0(\theta)$. Then, we have to evaluate four constants K_h which are dependent on four initial conditions.

If we define a vector K as $K = (K_1, K_2, K_3, K_4)^T$, we can have (12) for $t = 0$ as

$$K = \{M_2(\Psi)\}^{-1} \cdot \{I(0) - F_0(\Psi)\}. \tag{25}$$

VII. Practical Application

We have studied an alternator defined by the following data:

$$\sigma_{af} = 0.12 \quad \sigma_{akd} = 0.10 \quad \sigma_{akq} = 0.15 \quad \sigma_{fkd} = 0.06$$
$$\delta_a = 1.00 \quad \delta_f = 0.33 \quad \delta_{kd} = 0.10 \quad \delta_{kq} = 0.20.$$

Using the above method, we have determined the values of

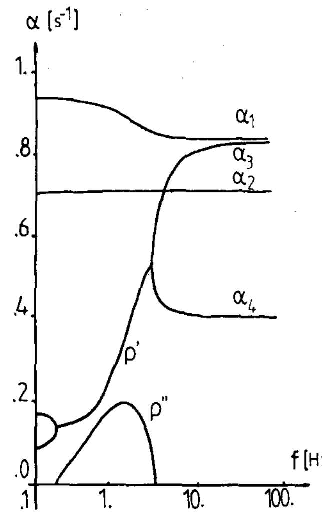

Fig. 2. Variation of time constants with frequency.

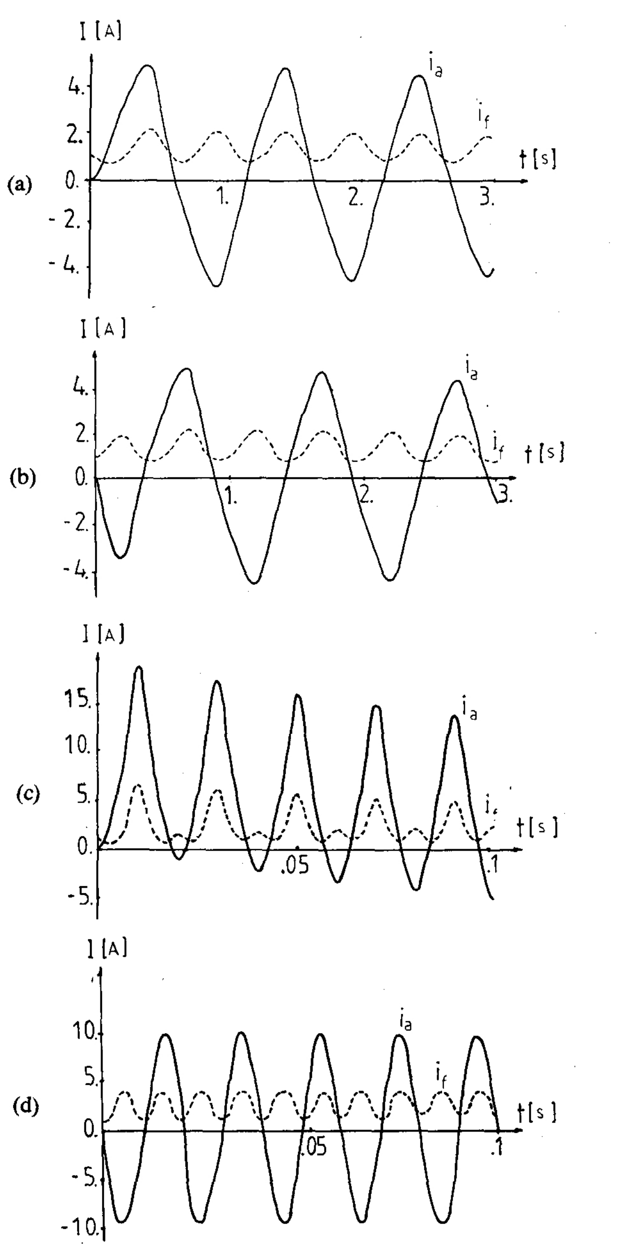

Fig. 3. Armature and field current variations: (a) $f=1$, $\Psi=0°$; (b) $f=1$, $\Psi=90°$; (c) $f=50$, $\Psi=0°$; (d) $f=50$, $\Psi=90°$.

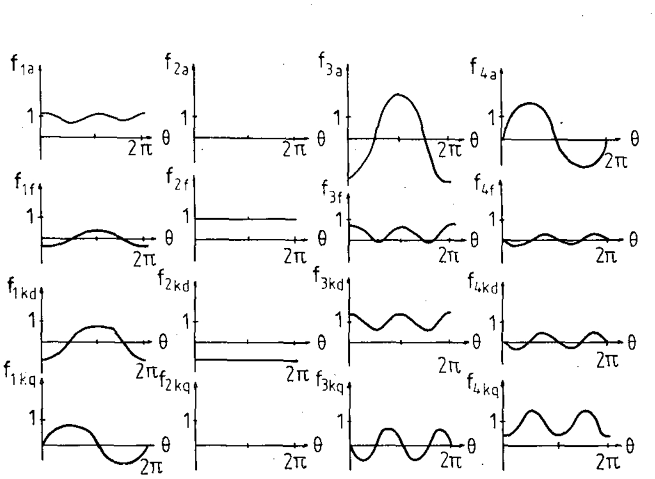

Fig. 4. The 16 periodic functions defining Floquet's reference frame.

TABLE I
MINIMUM VALUES OF J TO FULFILL VARIOUS CONDITIONS AT 0.1 Hz, 1.0 Hz, AND 50.0 Hz

f [Hz]	J_1	J_2	J_3
0.1	6	8	16
1.0	2	8	18
50.	1	8	20

α, displayed in Fig. 2. Those values are real and practically constant for $f < 0.1$ Hz and $f > 30$ Hz; in the interval $0.18 \leqslant f \leqslant 2.8$ Hz, two time constants are complex conjugate, and they are called $\rho' \pm j\rho''$. Then, the short-circuit currents have been computed for many values of the frequency and of Ψ, which is the initial value of θ. In each case, we have compared the results with the results yielded by a Runge-Kutta integration method. They tallied within reasonable roundoff errors; typically, the relative error was 10^{-5} for the peak values. Variations of i_a and i_f for $f=1$ Hz and $\Psi=0°$ and $\Psi=90°$ are given in Fig. 3(a) and (b); for $f=50$ Hz, Fig. 3(c) and (d) corresponds, respectively, to $\Psi=0°$ and $\Psi=90°$. The matrix $M_2(\theta)$, which is called Floquet's reference frame, is then real; it is described by the 16 curves in Fig. 4. The optimal value of J has been numerically studied. For each value of f, there is a good value of J, say J_1, which ensures values of α's within a 10^{-3} relative error. However, to determine correctly the currents, it is necessary to increase J from J_1, to J_2. Eventually, an attempt may be made to check the values of M_1, M_2, and M_3 as

$$M_3 = M_2^{-1}\left\{M_1 \cdot M_2 - \frac{d}{dt}M_2\right\}. \tag{26}$$

This is not so unless J is again increased to a larger value J_3. Values of J_1, J_2, and J_3 for $f=0.1$ Hz, $f=1.0$ Hz, and $f=50.0$ Hz are shown in Table I.

The following remarks can be made about Table I. First, J_1 increases when f decreases because, for $f=50$ Hz, matrices N_{-1} and P_1 are practically equal to $N_{-\infty}$ and P_∞, because α always appears in expressions $\alpha - jn\omega$. This is true for any other high value of frequency, but not for small values of frequency. Then, the fact that J_2 is independent of frequency means that the number of harmonics does not depend on frequency. Eventually, J_3 increases with frequency because of (26), where dM_2/dt increases with frequency, in such a way that accuracy seems to decrease with frequency.

As for the convergence of iterations, it is rather fast. For frequencies larger than 10 Hz, three iterations are enough to solve (20), provided that the initial guesses are chosen as explained above. For the lower frequencies (down to 0.1 Hz for an industrial machine), ten iterations may be necessary, and it is necessary to study successively several values of the frequency. For each value of frequency, the initial guess is the result obtained for the previously studied frequency.

VIII. Remarks

For an actual machine, the values of the σ's must lie in the interval $(0,1)$. For the sake of curiosity, we have used values outside this interval. The equation obtained in that way no longer represented the behavior of an electrical machine, but we have seen that the method still yields a correct result. Therefore, the success of the method is not limited to actual alternators, and it may be possible to extend it to a general stability problem [10], [11].

IX. Conclusions

We have found the Floquet reference frame for the single-phase short circuit of a uniform air gap synchronous machine with dampers by formulating the problem in a matrix form which is suitable for generalization.

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