NON LINEAR GROWTH CURVES FOR WEIGHT AND HEIGHT IN FOUR GENETIC GROUPS OF HORSES

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ABSTRACT

Hanoverian, Brazilian Showjumper, English Thoroughbred and Crossbred horses reared by the Brazilian Army were weighed and measured from six months of age to adult. In total 4,860 measurements on 1,445 horses were available. General curves were estimated as a function of time by the Gompertz, Brody, Logistic, Weibull and Richards curves, using PROC NLIN procedures of SAS ®. The Richards Curve did not converge for weight or height of any of the genetic groups or sexes. The logistic curve did not converge for any of the weight traits while the Gompertz also did not converge for height in several groups. R^2 varied between 0.55 for weight in females of the crossbred group to 0.92 for males of the same group. For the height traits the highest R^2 (0.66) was found for female Hanovarian horses and lowest for males of the same breed (0.12). In general the curves estimated similar values for asymptotic height and weight, except for Logistic curve, which also showed lowest R^2 and highest error. Results for the Weibull and Brody curves were similar in all cases so where possible the Brody curve was selected as the best curve as it had less parameters. The Gompertz curve tended to underestimate mature weights and height. Estimates for both weight and height were in general higher in males than for females. In most cases the b parameter was shown to account for < 0.0001% of the variation in the curve shape. The k parameters, which indicate maturity, were of similar magnitude for the Brody, Gompertz and Weibull curves, for both height and weight within breed. This parameter indicated that there is little difference in maturation rates between males and females.

KEY WORDS: Brody, Gompertz, Logistic, Richards, Weibull.

RESUMO

CURVAS DE CRESCIMENTO NÃO-LINEARES PARA PESO E ALTURA EM QUATRO GRUPOS GENÉTICOS DE CAVALOS

Cavalos das raças Hanoveriana (HA), Brasileiro de Hipismo (BH), Puro Sangue Inglês (PSI) e mestiços (PSIxBH) criados pelo Exército brasileiro foram pesados e medidos de seis meses de idade até adulto. Realizaram-se 4.860 medidas em 1.445 cavalos. Estimaram-se curvas de crescimento gerais usando os modelos de Gompertz, Brody, Logistic, Weibull e Richards, segundo o procedimento PROC NLIN do programa SAS ®. A curva de Richards não convergiu para peso nem altura para nenhum dos grupos ou sexos avaliados. A curva logística não convergiu para os pesos, enquanto o modelo de Gompertz não convergiu para altura em vários grupos. R^2 variou entre 0,55 para peso em fêmeas mestiças até 0,92 em machos do mesmo grupo. Para altura, o maior R^2 (0,66) foi para machos Hanovarianos e o menor para fêmeas da mesma raça (0,12). Em geral, as várias curvas estimaram a mesma altura e peso adulto, exceto a curva logística, que teve o menor R2 e mais alto erro dentro de cada grupo. Resultados para as curvas de Weibull e Brody foram similares em todos os casos. A curva de Brody foi selecionada como a melhor, porque possui menos parâmetros. A curva de Gompertz teve a tendência de subestimar pesos e alturas adultos. Estimativas para ambas as características foram mais altas em machos que fêmeas. No maior parte dos casos, o parâmetro b levou em consideração menos que 0,0001% da variação em forma da curva. Os parâmetros k, indicando maturidade, foram de magnitude similar para as curvas de Brody, Gompertz e Weibull, para altura e peso dentro de raça, o que indicou pouca diferença entre machos e fêmeas para taxa de maturação entre sexos.

PALAVRAS-CHAVES: Brody, Gompertz, Logistic, Richards, Weibull.

INTRODUCTION

Little is known about growth rates of horses and tracking growth means that informed decisions can be taken about growing horses' nutrition program and accurate assessments growth progress can be made. Rapid and/or irregular growth rates may be linked to the incidence of developmental orthopedic diseases and therefore growth tracking and accurate diet formulation are useful (KEELE et al., 1992; WILLIAMS et al., 1992) to reduce the on-farm incidence of developmental diseases (WEBSTER et al., 1982).

Modeling growth using mathematical functions summarizes growth data for an individual or a population (BATHAEI & LEROY, 1996). Many data points taken over time are reduced to a few parameters. The resulting curve provides a visual description of growth, and interpretation of the estimated parameters provides an explanation of what is occurring biologically (KSHIRSAGAR & SMITH, 1995).

The shape of growth curves have been reported to vary according to the species of animal, environment and trait (EFE, 1990; AKBAS et al., 1999). An ideal equation is one which adequately predicts the overall shape of a growth curve and, in addition, can be extended or modified to give greater flexibility and precision (MOORE, 1985). Selection of fast or slow growing animals can be carried out using these functions, especially animals which are more mature at an earlier age (BROWN et al., 1972; FITZHUGH, 1976).

Non linear functions with exponential components are most commonly used to describe this type of growth, as these summarize a large volume of data. These functions are easy to interpret biologically and easily compared between different production systems (SILVA et al., 2002). According to TEDESCHI et al. (2000), the parameters of nonlinear curves (such as Gompertz, Brody, Logistic, Weibull and Richards) which have biological interpretation are superior asymptotic (mature) weight and maturing rate, which is an indication of growth rate. The other parameters are mathematical constants which help to determine curve shape. The objective of this study was to select a non-linear function which best describes growth of horses reared by the Brazilian Army.

MATERIAL AND METHODS

All horses in the Brazilian Army are produced by the Coudelaria de Rincão, located in São Borja, Rio Grande do Sul State. It is situated at latitude 55° 35' 00" south and 28° 45' 40" West, with an altitude of 130m and the climate is Humid subtropical Cfa according to the Köppen classification, with rainfall well distributed throughout the year and mean 1,350 mm. The study has 206 dams and 15 stallions. Both mares and stallions were from the proper herd or on loan from other studs. Weaning was at six months of age. Pastures were based on temperate grasses including oats (*Avena sp*), azevém (*Lolium multiflorum*) and clover (*Trifolium repens*). Stallions also received alfalfa hay daily (*Medicago sativa*) *ad libitum*.

Data were also collected from Military Organizations distributed in all of Brazilian national territory, except Amazon, where the animals are sent after 24 months of age. In these installations they receive concentrate with 14% crude protein (CP), divided in three meals a day and foals received a ration with 18% CP. The horses may be stabled or semi-stabled, depending on the station. The horses may be transferred between stations, depending on necessity, or because they are used by a particular officer who uses the horse in competition. Data was collected by veterinarians in each station.

Data was available on 4,860 weight and shoulder height measurements on 1,445 Hanoverian, Brazilian Showjumper, English Thoroughbred and Crossbred horses, measured from six months of age to adult.

Curves

Weight and height were analysed as a function of time using Gompertz, Brody, Logistic, Weibull and Richards curves. The Richards curve describes changes in size Y (weight or height) in relation to age t, and is represented by the equation:

$$Y_t = a (1 \pm be^{-kt})^m.$$

The interpretations of the parameters are as follows: a = asymptotic value interpreted as mean weight/height at maturity; b = scale parameter (integration constant), indicates the proportion of asymptotic growth to be gained after birth, and is established by initial values of Y and t; k = index of maturity or precocity

measure. Calculating k^{-1} the time used to obtain maturity is found, which serves as a measure of maturity (TAYLOR & CRAIG, 1965). The greater the value of k means that animals mature earlier; m = is inflexion parameter, which establishes the degree of maturity u at the inflexion point $u_i = [(m-1)/m]^m$, with u_t the proportion of weight attained at age t: $u_t = Y_t/a = (1-be^{-kt})^m$; and e is the Napier base for natural logarithms. The sign "$\pm$" when $m \geq 1$, is – and + when $m < 0$.

The parameter m assumes the following values for the other curves in the Richards family: Brody (1), Gompertz ($\mu \to \infty$) and Logistic (-1). The Brody and Logistic curves are respectively, $Y_t = a-be^{-kt}$ and $Y_t = a(1+be^{-kt})^{-1}$. The Gompertz curve is a limiting case of the generalised logistic as t becomes very small or very large, whose equation is: $Y_t = a\,e^{-be^{\wedge -kt}}$. The Weibull curve is : $Y_t = a- (be^{-kt^{\wedge}d})$

Procedures described by PEREIRA & ARRUDA (1987) and SAS (1987) were used for convergence of non-linear data. The values of sum of squares of the residual (SQr), coefficient of determination (r^2) and divergence from regression for each function were calculated. The r^2 was calculated as $r^2 = 1 (sqe)/(sqt)$, where sqe is the sum of squares of the error and sqt, is the total sum of squares (KVÅLSETH, 1985).

Individual estimates of growth parameters were obtained using the modified Gauss-Newton method in the NLIN procedure of SAS ® (HARTLEY, 1961; LAWTON et al., 1972). Convergence was assumed when the difference in sum of residual sum of squares between the $i^{th} -1$ to the ith iteration was < 10^{-8}. The mean square of the residual was calculated for each function. The r^2 and error were used to evaluate the adjust of the curve.

RESULTS

The Richards Curve did not converge for weight or height of any of the genetic groups or sexes. The logistic curve did not converge for any of the weight traits while the Gompertz also did not converge for height in several groups. R^2 varied between 0.55 for weight in females of the crossbred group to 0.92 for males of the same group. For the height traits the highest R^2 (0.66) was found for female Hanovarian horses and lowest for males of the same breed (0.12). Tables 1 to 4 shows a summary of the analyses carried out for each breed and trait. Figures 1 and 2 show the best fit curves for the data in this study with 95% confidence limits. Adult height and weight is reached in all cases at about three years of age.

TABLE 1. Growth curve parameters for height and weight of Brazilian Showjumpers in the Brazilian Army

Curve	Weight				Height			
	Weibull	Brody	Gompertz	Logistic	Weibull	Brody	Gompertz	Logistic
				Males				
R^2	0.637	0.637	0.628	-	0.603	-	-	-
Error	48.588	47.992	49.186	-	0.051	-	-	-
A	517.026	517.021	511.361	-	1.623	-	-	-
B	411.902	0.797	0.026	-	0.257	-	-	-
K	0.003	0.002	0.002	-	0.002	-	-	-
D	0.683	-	56.767	-	0.508	-	-	-
M	-	-	-	-	-	-	-	-
				Females				
R^2	0.631	0.631	0.635	-	0.551	0.551	0.551	0.095
Error	49.058	49.012	48.808	-	0.053	0.053	0.053	0.076
A	465.706	465.706	175.826	-	1.568	1.568	-27.884	1.501
B	716.427	1.538	283.711	-	0.590	0.376	29.452	0.001
K	0.004	0.004	0.006	-	0.004	0.004	0.004	0.018
D	1.020	-	326.755	-	0.098	-	-998.910	-24.344
M	-	-	-	-	-	-	-	37.162

R^2 – determination coefficient; a,b,k,d,m – see text for details

TABLE 2. Growth curve parameters for height and weight of Hanoverian Horses in the Brazilian Army

	Weight				Height			
	Weibull	Brody	Gompertz	Logistic	Weibull	Brody	Gompertz	Logistic
				Male				
R2	0,631		0,635		0,551		0,551	0,095
Error	49,058		48,808		0,053		0,053	0,076
A	465,706		175,826		1,568		-27,884	1,501
B	716,427		283,711		0,590		29,452	0,001
K	0,004		0,006		0,004		0,004	0,018
D	1,020		326,755		0,975		-998,910	-24,344
M								37,162
				Female				
R^2	0,695	0,695	0,696		0,581	0,581	0,581	0,581
Error	40,309	40,268	40,232		0,048	0,047	0,048	0,475
A	469,036	469,036	395,455		1,563	1,563	-25,824	1,583
B	638,776	1,362	68,576		0,593	0,379	27,387	-0,019
K	0,004	0,004	0,005		0,004	0,004	0,004	0,003
D	0,970		227,678		0,979		-971,133	-5,392
M								-453,299

R^2 – determination coefficient; a,b,k,d,m – see text for details

TABLE 3. Growth curve parameters for height and weight of English Thoroughbred Horses in the Brazilian Army

	Weight				Height			
	Weibull	Brody	Gompertz	Logistic	Weibull	Brody	Gompertz	Logistic
				Male				
R^2	0,594	0,594	0,594		0,128	0,120	0,371	0,198
Error	32,217	31,357	32,193		0,065	0,064	0,054	0,064
A	407,668	407,668	317,266		1,524	1,526	1,613	1,905
B	486,156	1,193	89,680		-173868,480	-59,150	0,110	-0,385
K	0,004	0,004	0,005		559,580	0,644	0,001	0,015
D	1,004		180,066		136482,940			15,164
M								55,419
				Female				
R^2	0,921		0,920		0,662			0,040
Error	15,800		15,910		0,048			0,083
A	287,481		286,646		1,563			1,502
B	1405,582		0,144		0,354			0,002
K	0,006		0,010		0,003			0,019
D	1,490		208,682		0,626			-101,87
M								36,964

R^2 – determination coefficient; a,b,k,d,m – see text for details

TABLE 4. Growth curve parameters for height and weight of No Defined Breed Horses in the Brazilian Army

	Weight				Height			
	Weibull	Brody	Gompertz	Logistic	Weibull	Brody	Gompertz	Logistic
				Male				
R^2	0,593	0,593	0,596		0,454	0,454	0,454	0,459
Error	47,985	47,967	47,857		0,059	0,059	0,059	0,058
A	447,532	447,532	131,606		1,543	1,543	-28,094	-2,631
B	570,498	1,275	310,446		0,506	0,328	29,637	4,182
K	0,004	0,004	0,005		0,004	0,004	0,004	0,002
D	0,963		273,614		0,947		-1102,628	-25,174
M								-1307,832
				Female				
R^2	0,555		0,560		0,517	0,517	0,517	0,519
Erro	49,679		49,416		0,050	0,050	0,050	0,050
A	438,519		261,845		1,538	1,538	-35,370	1,050
B	630,423		172,221		0,532	0,346	36,909	2,591
K	0,004		0,006		0,004	0,004	0,004	0,003
D	1,028		311,203		0,978		11077,448	-10,685
M								-709,391

R^2 – determination coefficient; a,b,k,d,m – see text for details

In general the curves estimated similar values for asymptotic height and weight, except for Logistic curve, which also showed lowest R^2 and highest error. Results for the Weibull and Brody curves were similar in all cases so where possible the Brody curve was selected as the best curve as it had less parameters. The Gompertz curve tended to underestimate mature weights and height. Estimates for both weight and height were in general higher in males than for females.

In most cases the *b* parameter was shown to account for < 0.0001% of the variation in the curve shape. The *k* parameters, which indicate maturity, were of similar magnitude for the Brody, Gompertz and Weibull curves, for both height and weight within breed. This parameter indicated that there is little difference in maturation rates between males and females.
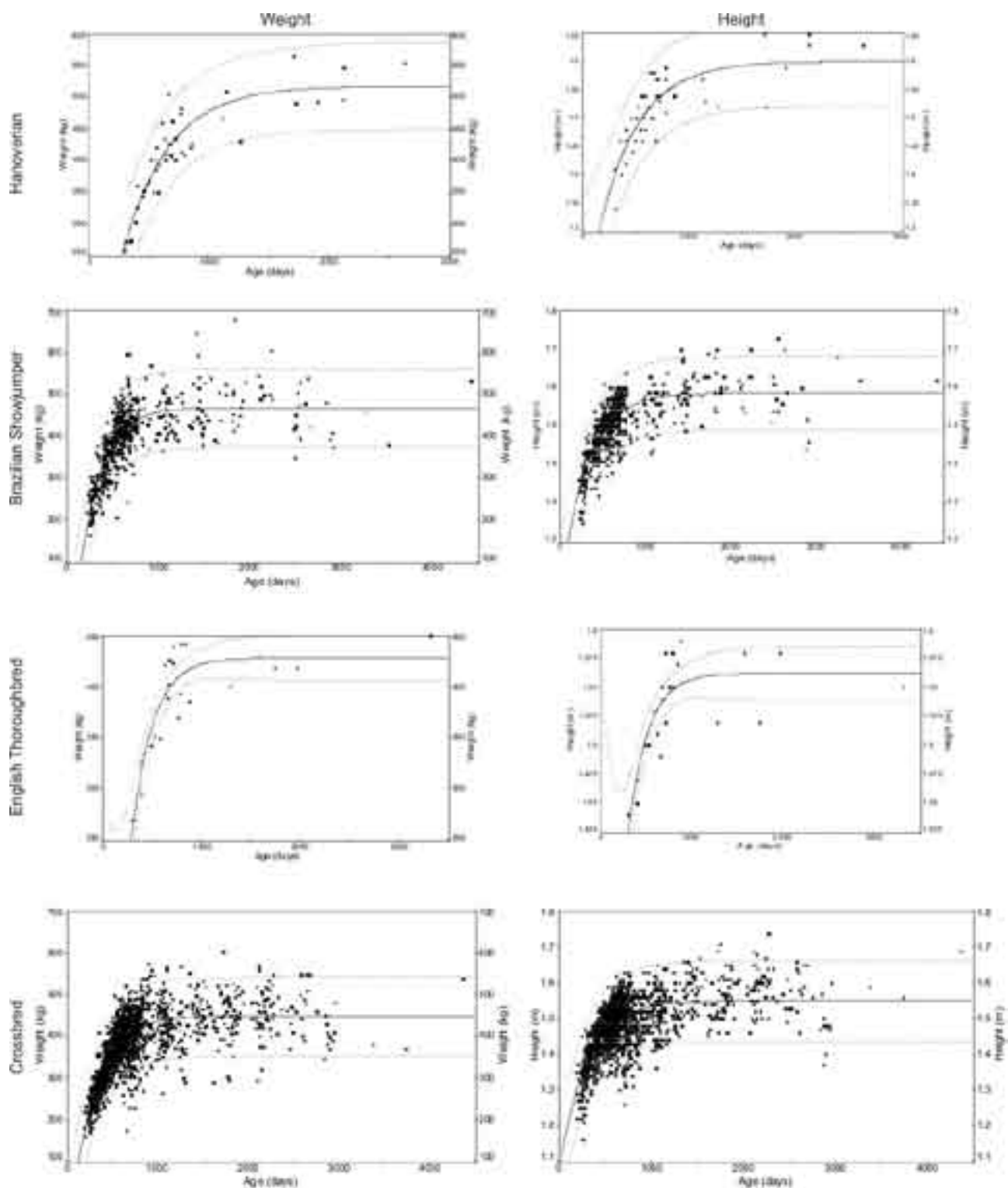
FIGURE 1. Weights and heights for male horses in the Brazilian Army with regression line and 95% confidence intervals.
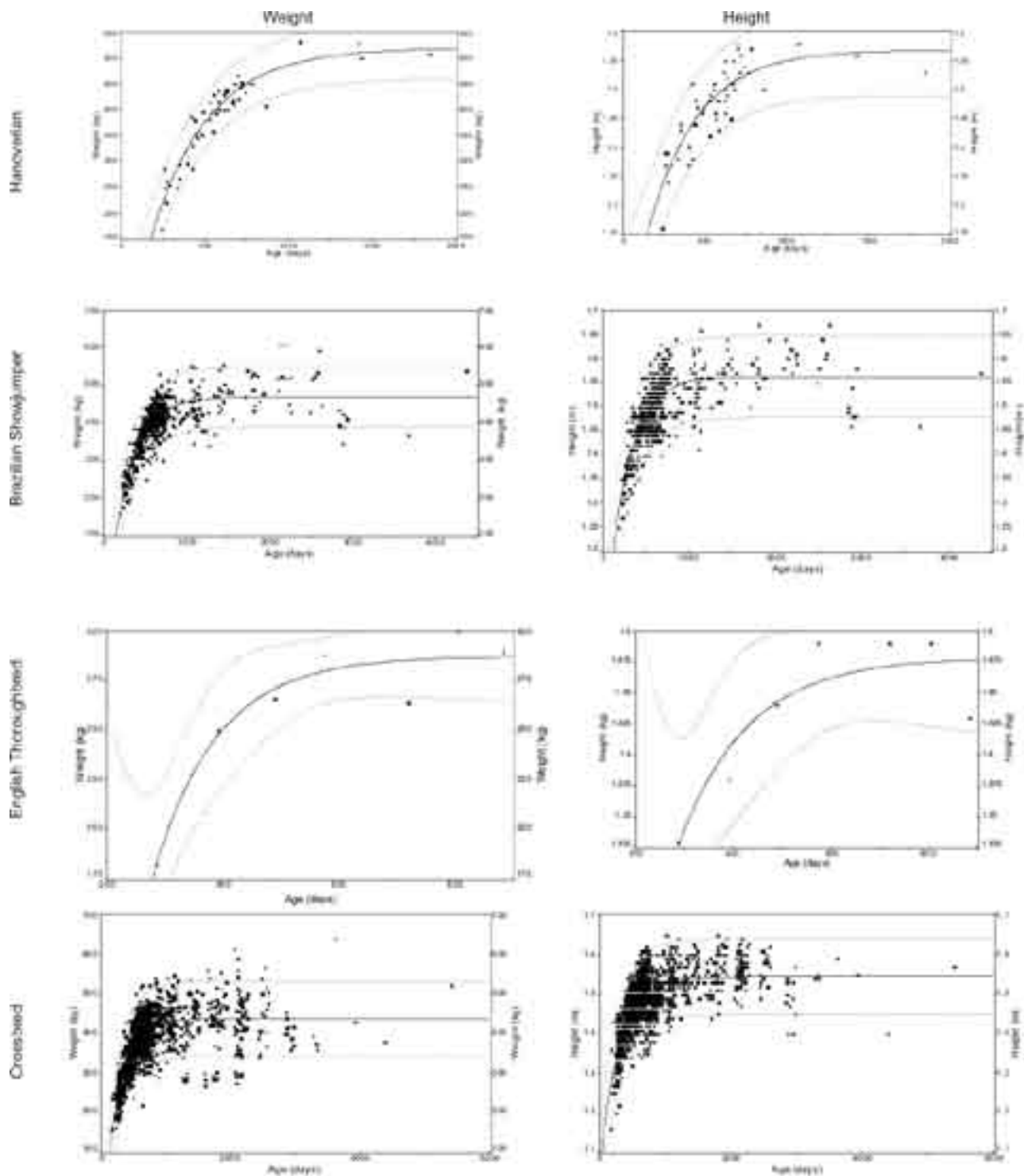
FIGURE 2. Weights and heights for female horses in the Brazilian Army with regression line and 95% confidence intervals.

DISCUSSION

Mean adult values estimated by the adjusted curves are very close to the mean values found by CAMPOS et al. (2007) in the adult horses (>48 months of age) of both sexes. The Hanoverian horse is a founder of the Brazilian Showjumper breed (DIAS et al., 2000), the two largest breeds in this study.

The logistic and the Gompertz equations have three parameters, all of which are biologically interpretable and statistically significant in the present analysis. Parameters having a straightforward meaning are advantageous for statistical parameterization of non-linear equations. Parameters of such non-linear functions have to be estimated by using an iterative regression approach, such as PROC NLIN of SAS ®, which requires an initial estimate of the parameters. Initial values here were taken from SANTOS et al. (2007) for Pantaneiro horses.

Contrary to this study, experiments have shown that the Richards function has the best for modeling this type of data (BROWN et al., 1976; DeNISE & BRINKS, 1985), but these studies also show that the Brody function is better above six months of age, as with these data, where the youngest age was 6 months. Other authors found that while the logistic function underestimates and Brody overestimates adult weight compared to the Richards and Von Bertalanffy functions (DUARTE, 1975; PEROTTO et al., 1992), although standardization has been carried out for use of the Brody function in cattle (BROWN et al., 1972; DUARTE, 1975; SILVEIRA JR., 1976; LUDWIG, 1977). Although it is shown to be flexibile, the Richards equation has often been criticized as the shape parameter has no obvious biological interpretation and is so unstable numerically that its estimate becomes useless (ZEIDE, 1993). YIN et al. (2003) found that in no case did the Richards equation achieve a statistically significant improvement over the logistic equation, and in only two cases did it improve on the Gompertz equation. For the data sets used in the present analysis the Richards equation did not converge. TEDESCHI et al. (2000) also found that the Richards curve had the most difficulty in converging when using Brazilian beef cattle data.

No mathematical model can accurately describe every biological phenomenon (TABATABAI et al., 2005). Many models have been developed to deal with sigmoid growth (ZEIDE, 1993) and new ones are continuously being proposed. While the logistic function is symmetric around the point of inflection, the Richards function is more flexible and can fit asymmetric growth patterns (TABATABAI et al., 2005; ZHU et al., 1998), however, it has more parameters than the logistic function. The Gompertz function has the same number of parameters as the logistic function and the Weibull function has the same number of parameters as the Richards function and both can fit asymmetric growth, but they are not very flexible (YIN et al., 2003). In the logistic model, the growth curve is symmetric around the point of maximum growth rate and has equal periods of slow and fast growth (TABATABAI et al., 2005). In contrast, the Gompertz model does not incorporate the symmetry restriction and has a shorter period of fast growth. Both the logistic and Gompertz have points of inflection that are always at a fixed proportion of their asymptotic population values. The same author noted that the description of growth by fixed-shape sigmoid models such as logistic, Gompertz or Von Bertalanffy curves may not be adequate because of the failure of the assumption that a constant curve shape holds across treatment groups. While the above equations predict a positive non-zero value for Y at time $t = 0$, the Weibull function predicts $Y_t = 0$ when t = 0 (YIN et al., 2003), which is not true with growing animals.

The results found here are different from those found by SANTOS et al. (2007) working with Pantaneiro horses and who chose Richards and Weibull curves for shoulder height and weight, respectively. SANTOS et al. (1998) with different data, also for the Pantaneiro horse, chose the Weibull curve for shoulder height. It also differs from the results found by FREITAS (2005), who studied growth curves in eight species of animal, and concluded that the logistic model was the most adequate. Figures 1 and 2 show the resultant curves and 95% confidence limits for male and female horses respectively.

The choice of the best curve using the analysis of variance residuals is not necessarily the best option as longitudinal data (such as weight and height on the same animal) show correlated errors between ages. These errors are caused by fluctuations over time, due

to various factors which are not necessarily reflected in the curve. Recommended methods for curve selection include the evaluation of the difference between observed and predicted values at specific ages (BROWN et al., 1976), but these ages are not always available. Other methods include: residual sum of squares (PEROTTO et al., 1992), regression deviations, determination coefficient (R^2), percentage and difficulty of convergence (BROWN et al., 1976; OLIVEIRA et al., 2000), curve behaviour and evaluation of parameters by comparison and graphic evaluation of the curves (FITZHUGH, 1976). Here R^2 and divergence were used as criteria for selection of the model.

Other authors (JONES, 1987; HEUSNER, 1992; YAMAMOTO et al., 1993; THOMPSON et al., 1994 and PAGAN et al., 1996), found that males generally presented higher growth rates than females. The males in this study were castrated at 24 months of age, but some authors (HEUSNER, 1997) state that castration does not affect development at this age.

Growth data in the literature is limited for adult horses of the breeds studied here. Adult shoulder height was found to be 137.65 cm for the Pantaneiro horse in Brazil (MISERANI et al., 2002) and 144cm for Campeiro (McMANUS et al., 2005). The same trait in Mangalarga Machador horses was found to be 151.5cm and 151.6cm for males and females respectively (CABRAL et al., 2004) and 149 and 150cm for Arabian horses (SADEK et al., 2006). Using regression equations, STAINER et al. (2004) showed that Thoroughbred horses reached 542 ± 6.2 kg reached at 7 yrs, somewhat heavier than found here.

CONCLUSION

The Weibull and Brody curves best fitted the increase of height and weight for horses reared in the Brazilian Army. Other curves returned results not consistent with the data or, in the case of the Richards curve, failed to converge. There was little difference between sexes for maturing rate.

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REFERENCES

AKBAS, Y.; TASKÝN, T.; DEMIRÖREN, E. Comparison of several models to fit the growth curves of Kivircik and Daglic male lambs. **Turkish Journal of Veterinary Animal Science**, v. 23, Suppl. 3, p. 537-544, 1999.

BATHAEI, S. S.; LEROY, P. L. Growth and mature weight of Mehraban Iranian fat-tailed sheep. **Small Ruminant Research**, v. 22, p. 155-162, 1996.

BATHAEI, S. S.; LEROY, P. L. Genetic and phenotypic aspects of the growth curve characteristics in Mehraban Iranian fat-tailed sheep. **Small Ruminant Research**, v. 29, p. 261-269, 1998.

BRACCINI NETO, J.; DIONELLO, N. J. L.; SILVEIRA JR. P.; BONGALHARDO, D. C.; XAVIER, E. G. Análise de curvas de crescimento de aves de postura. **Revista Brasileira de Zootecnia**, v. 25, n. 6, p. 1062-1073, 1996.

BROWN, J. E.; FITZHUGH Jr. H. A.; CARTWRIGHT, T. C. A comparison of nonlinear models for describing weight–age relationships in cattle. **Journal of Animal Science**, v. 42, p. 810-818, 1976.

CABRAL G. C.; ALMEIDA F. Q. de; QUIRINO C. R.; PINTO, L. F. B.; SANTOS, E. M.; CORASSA, A. Morphmetric evaluation of Mangalarga Marchador horses: linear measures. **Revista Brasileira de Zootecnia**, v. 33, p. 989-1000, 2004.

DeNISE, R. S. K.; BRINKS, J. S. Genetic and environmental aspects of the growth curve parameters in beef cows. **Journal of Animal Science**, v. 61, p. 1431-1440, 1985.

DUNCAN, D. B. A multiple range and multiple F tests. **Biometrics**, v. 2, p. 1-42, 1955.

EFE, E. B**üyüme eðrileri**. 1990. Ph.D. (Thesis) – Çucurova. Üniversitesi, Adana, 1990.

ELIAS, A. M. Análise de curvas de crescimento de vacas das Raças Nelore, Guzerá e Gir. 1998. 128 f. Dissertação (Mestrado em Ciência Animal e Pastagens) – ESALQ, 1998.

ESENBUGA, N.; BILGIN, Ö. C.; MACIT, M.; KARAOGLU, M. Ivesi, Morkaraman ve Tuj kuzularinda Büyüme Ergileri (Growth curves in Awassi, Morkaraman and Tushin Lambs). **Atatürk Üniversitesi Ziraat Fak. Derg**, v. 31, p. 37-41, 2000.

FITZHUGH Jr. H. A. Analysis of growth curves and strategies for altering their shapes. **Journal of Animal Science**, v. 42, p. 1036-1050, 1976.

GOONEWARDENE, L. A.; BERG, R. T.; HARDIN, R. T. A growth study of beef cattle. **Canadian Journal of Animal Science**, v. 61, p. 1041-1048, 1981.

HALD, A. **Statistical theory with engineering applications**. New York: Wiley, 1952. p. 658-662,

HELMINK, S. K.; SHANKS, R. D.; LEIGHTON, E. A. Breed and sex differences in growth curves for two breeds of dog guides. **Journal of Animal Science**, v. 78, p. 27-32, 2000.

KOCABAS, Z.; KESICI, T.; ELIÇIN, A. Growth curve in Akkaraman, Awassi×Akkaraman and Malya×Akkaraman lambs. **Turkish Journal of Veterinary Animal Science**, v. 21, p. 267-275, 1997.

KROLL, L. B. Estudo do crescimento de vacas leiteiras através de modelos com autocorrelação nos erros. **Revista Multiciência**, v. 1, p. 164-165, 1996.

KSHIRSAGAR, A. M.; SMITH, W. B. **Growth curves**. New York: Marcel Dekker, 1995. p. 1-57

KVÅLSETH, T. O. Cautionary note about R². **The American Statistician**, v. 39, p. 279-285, 1985.

LEWIS, R. M.; EMMANS, G. C.; DINGWALL, W. S.; SIMM, G. A description of the growth of sheep and its genetic analysis. **Animal Science**, v. 74, p. 51-62, 2002.

TOPAL, M.; OZDEMIR, V.; AKSAKAL, N.; YILDIZ, V.; DOGRU, U. Determination of the best nonlinear function in order to estimate growth in Morkaraman and Awassi lambs **Small Ruminant Research**, v. 55, p. 229-232, 2004.

McMANUS, C.; FALCÃO, R. A.; SPRITZE, A.; COSTA, D.; LOU-VANDINI, H.; DIAS, L. T.; TEIXEIRA, R. A.; REZENDE, M. J. M.; GARCIA, J. A. S. Caracterização morfológica de equinos da raça campeiro. **Revista Brasileira de Zootecnia**, v. 34, p. 1553-1562, 2005.

MEDEIROS, H. A.; AQUINO, L. H.; BEARZOTI, E.; FERREIRA, D. F. Avaliação da qualidade do ajuste da função logística monofásica com estrutura de erros independentes e autoregressivos através de simulação. **Ciência e Agrotecnologia**, v. 24, p. 973-985, 2000.

MISERANI, M.; McMANUS, C.; SANTOS, S. A.; SILVA, J. A.; MARIANTE, A. S.; ABREU, U. G. P.; MAZZA, M. C.; SERENO, J. R. B. Variance analysis for biometric measures of the Pantaneiro Horse **Archivos de Zootecnia**, v. 51, p. 113-120, 2002.

MOORE, A. J. A mathematical equation for animal growth from embryo to adult. **Animal Production**, v. 40, p. 441-453, 1985.

MORETTIN, P. A.; TOLOI, C. M. de C. **Análise de séries temporais**. São Paulo: Edgard Blücher, 2004. 536 p.

NETER, J.; WASSERMAN, W.; KUTNER, M. H. **Applied linear regression models**. Boston: Irwin, 1989. p. 550-562.

OLIVEIRA, H. N.; LÔBO, R. B.; PEREIRA, C. S. Comparação de modelos não-lineares para descrever o crescimento de fêmeas da raça Guzerá. **Pesquisa Agropecuaria Brasileira**, v. 35, p. 1843-1851, 2000.

PASTERNAK, H.; SHALEV, B. A. The effect of a feature of regression disturbance on the efficiency of fitting growth curves. **Growth, Development & Aging**, v. 58, p. 33-39, 1994.

SADEK, M. H.; AL-ABOUD, A. Z.; ASHMAWY, A. A. Factor analysis of body measurements in Arabian horses. **Journal of Animal Breeding and Genetics**, v. 123, p. 369-377, 2006.

SANTOS, R. dos. Os cruzamentos na pecuária tropical. **Agropecuária Tropical**, 1999. 672 p.

SAS Institute. **SAS/ETS® User's Guide.** Version 6. 2. ed. Cary: SAS Institute, 1995. 373 p.

SILVA, F. F. E.; de AQUINO, L. H.; de OLIVEIRA, A. I. G. Estimativas de parâmetros genéticos em curvas de crescimento de gado Nelore. **Ciência Agrotecnica**, v. 26, supl., p. 1562-1567, 2002.

SOUZA, G. da S. **Introdução aos modelos de regressão linear e não linear**. Brasília: Embrapa-SPI/Embrapa-SEA, 1998. 489 p.

STANIAR, W. B.; KRONFELD, D. S.; TREIBER, K. H.; SPLAN, R. K.; HARRIS, P. A. Growth rate consists of baseline and systematic deviation components in Thoroughbreds. **Journal of Animal Science**, v. 82, p. 1007-1015, 2004.

TABATABAI, M.; WILLIAMS D. K.; BURSAC, Z. Hyperbolastic growth models: theory and application **Theoretical Biology and Medical Modelling**, v. 2, p. 14, 2005.

TEDESCHI, L. O.; BOIN, C.; NARDON, R. F.; LEME, P. R. Estudo da curva de crescimento de animais da raça guzerá e seus cruzamentos alimentados a pasto com e sem suplementação. 1. Análise e seleção das funções não-lineares. **Revista Brasileira de Zootecnia**, v. 29, p. 630-637, 2000.

YIN, X.; GOUDRIAAN J.; LATINGA E. A.; VOS, J.; SPIERTZ, H. J. A flexible sigmoid function of determinate growth. **Annals of Botany**, v. 91, p. 361-371, 2003.

ZEIDE, B. Analysis of growth equations. **Forest Science**, v. 39, p. 594-616, 1993.

ZHU, Q.; CAO, X.; LUO, Y. Growth analysis on the process of grain filling in rice. **Acta Agronomica Sinica**, v. 14, p. 182-193, 1988.